Int. J. Heat Mass Transfer. Vol. 6, pp. 483–493. Pergamon Press 1963. Printed in Great Britain.

HEAT TRANSFER IN ANNULAR PASSAGES. GENERAL FORMULATION OF THE PROBLEM FOR ARBITRARILY PRESCRIBED WALL TEMPERATURES OR HEAT FLUXES

W. C. REYNOLDS*, R. E. LUNDBERG† and P. A. McCUEN‡

(*Received* 19 *August and in revised form* 12 *December* 1962)

Abstract—This paper represents the first in a series culminating a four-year study§ of heat transfer in annular passages. A general formulation is developed herein, and the contributions of the several phases of the program outlined. Apparatus used in experimental evaluation of annulus heat-transfer functions is described, and an extensive bibliography of annulus literature is included.

NOMENCLATURE

A, area, ft^2;
c_p, specific heat at constant pressure, Btu/lb degF;
D_h, flow passage hydraulic diameter, ft;
k, thermal conductivity, Btu/h ft degF;
q'', heat flux, Btu/h ft^2;
r, radial co-ordinate, ft;
Re, Reynolds number, $U_m D_h/\nu$ dimensionless;
S, circumferential co-ordinate, ft;
t, T, temperature, °F;
U, x velocity component, ft/s;
U_m, mixed mean velocity, ft/s;
V, r velocity component, ft/s;
W, angular velocity component, ft/s;
x, axial co ordinate, ft.

Greek symbols

α, thermal diffusivity, $k/\rho c_p$, ft^2;
δ, displacement of axes, ft;
ϵ_{Hr}, eddy diffusivity for heat in the r-direction, ft^2/s;
$\epsilon_{H\Theta}$, eddy diffusivity for heat in the Θ-direction, ft^2/s;
$\Phi_{ij}^{(k)}$, heat flux function, dimensionless;
$\theta_{ij}^{(k)}$, wall temperature function, dimensionless;
$\theta_{mj}^{(k)}$, mean temperature function, dimensionless;
Θ, angular co-ordinate, radius;
ρ, density, lb/ft^3;
ν, kinematic viscosity, ft^2/s;
ξ, dummy axial co-ordinate, ft.

* Associate Professor, Stanford University, Stanford, California.
† Research Engineer, Stanford Research Institute, Menlo Park, California.
‡ Staff Scientist, Vidya, Inc., Palo Alto, California.
§ The support of the National Aeronautics and Space Administration is greatfully acknowledged.

INTRODUCTION

THE annulus represents a common geometry employed in a variety of heat-transfer systems ranging from simple heat exchangers to the most complicated nuclear reactors. While there have been numerous studies of both laminar and turbulent heat transfer in annuli, in general previous efforts have been rather restrictive in their scope, in terms of the geometrical parameters, working fluids, and thermal boundary conditions. This paper is the first of a series which represents the culmination of an extensive four year study of annulus heat transfer at Stanford University [1, 2, 3, 4, 5]. It is designed to serve as a basis of departure for subsequent phases of the program. Here we formulate the problem of annulus heat transfer in general, outline the contributions of each of the program phases, and describe experimental apparatus common to several annulus experiments. No detailed review of the literature will be made, but a bibliography of pertinent publications is included here. Previous contributions to each phase of the program will be described in the subsequent papers.

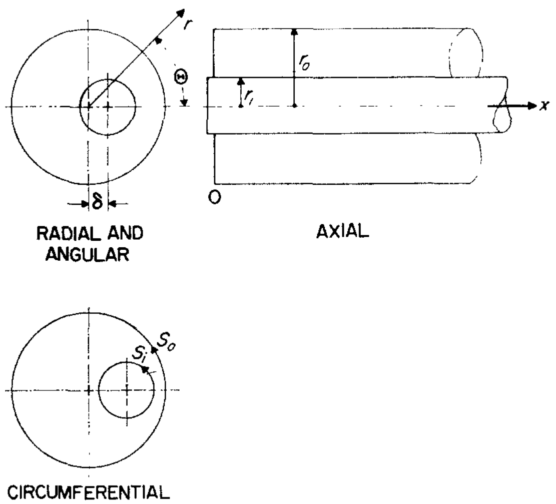

FIG. 1. Co-ordinates.

FORMULATION

The co-ordinate system used throughout is shown in Fig. 1. The point $x = 0$ will be used to designate the entrance to the annulus, where a uniform velocity profile is assumed to exist. Velocity components in the x, r, and Θ directions will be denoted by U, V, and W, respectively. We consider impermeable walls, and in general U, V, and W will be functions of x, r, and Θ. For the concentric case ($\delta = 0$) W will be zero, and U and V will be independent of Θ. For hydrodynamically developed flow, V and W will be zero, and U will be independent of x.

Although the hydrodynamics is an essential part of any convective thermal analysis, the velocity fields employed in analyses in subsequent parts are obtained in decidedly different manners. For the purpose of development of the general convective heat-transfer approach we will consider that the velocity field is known, steady, and independent of the thermal field. In addition, for the case of turbulent flow, we assume that eddy diffusivities for turbulent heat transfer in the r and Θ directions, ϵ_{Hr} and ϵ_{H_Θ}, defined in the usual way, can be evaluated from the turbulent velocity field, and are independent of temperature. We neglect both eddy and molecular conduction in the flow direction. With the additional idealizations that the temperature field is independent of time, that enthalpy variations are due only to temperature changes, and that the thermal and transport properties of the fluid are constant, and neglecting conversions of energy to thermal forms within the fluid, the differential equation describing the temperature field may readily be obtained from energy and mass balances on a differential control volume of the r-Θ-x space, and is

$$\frac{\partial}{\partial r}\left[r(\epsilon_{Hr} + \alpha)\frac{\partial T}{\partial r}\right] + \frac{\partial}{\partial \Theta}\left[(\epsilon_{H_\Theta} + \alpha)\frac{\partial T}{\partial \Theta}\right] = Ur\frac{\partial T}{\partial x} + Vr\frac{\partial T}{\partial r} + W\frac{\partial T}{\partial \Theta}. \tag{1}$$

This equation is considerably too general for a tractable treatment of annulus heat transfer. However, we wish to develop a very general superposition approach, in terms of four kinds of fundamental solutions, which may be obtained analytically or experimentally. Analytical solutions, obtained after further simplifications, are included in subsequent parts.

THE FUNDAMENTAL SOLUTIONS

The idealizations described above imply that the coefficients in (1) are independent of temperature; consequently (1) is linear and homogeneous in T, and the sum of multiples of any solutions will also be a solution. We can, therefore, obtain rather complex solutions by adding together a variety of simpler solutions. Upon reflection it appears that with four kinds of fundamental solutions most types of boundary conditions can be handled. While the following development can be extended directly to cases where circumferential variations in wall temperature or heat flux are prescribed, or where the inlet temperature profile is non-uniform, we will, in the interests of clarity, omit these possibilities and consider only fundamental solutions necessary for handling problems where the inlet temperature is uniform and the prescribed wall boundary conditions are independent of the circumferential co-ordinates S_i and S_o. The fundamental solutions are all dimensionless, and the reader will perhaps find it convenient to replace t by θ in (1), where $\theta(x, r, \Theta)$ represents a dimensionless field. Dimensional temperature fields will be obtained later by multiplication of the fundamental solutions by appropriate constants.

1. *Fundamental Solutions of the First Kind*

We let $\theta_j^{(1)}(x, r, \Theta; \xi)$ denote the solution of (1) satisfying the boundary conditions

$$\theta_j^{(1)} = 0, \quad \begin{cases} \text{(a) on wall } j \text{ for } x < \xi \\ \text{(b) on opposite wall for all } x \\ \text{(c) everywhere upstream of } \xi \end{cases}$$

$$\theta_j^{(1)} = 1, \quad \text{(a) on wall } j \text{ for } x \geqslant \xi$$

$\theta_j^{(1)}$ represents a solution corresponding to a step temperature on one wall, with the other wall maintained at the inlet temperature, and henceforth will be called *the fundamental solution of the first kind.**

2. *Fundamental Solutions of the Second Kind*

We let $\theta_j^{(2)}(x, r, \Theta; \xi)$ represent the solution of (1) satisfying the boundary conditions

$$\theta_j^{(2)} = 0, \quad \text{(a) everywhere upstream of } \xi$$

$$\Phi_j^{(2)} = -D_h \frac{\partial\theta_j^{(2)}}{\partial n}$$

$$= \begin{cases} 0 \begin{cases} \text{(a) on wall } j \text{ for } x < \xi \\ \text{(b) on opposite wall for all } x \end{cases} \\ 1 \quad \text{(a) on wall } j \text{ for } x \geqslant \xi \end{cases}$$

$\theta_j^{(2)}$ represents a solution corresponding to a step heat flux at ξ from one wall only, the other wall being kept adiabatic, and henceforth will be called *the fundamental solution of the second kind.*

3. *Fundamental Solutions of the Third Kind*

We let $\theta_j^{(3)}(x, r, \Theta; \xi)$ represent the solution of (1) satisfying the boundary conditions

$$\theta_j^{(3)}, \quad = \begin{cases} 0 \begin{cases} \text{(a) everywhere upstream of } \xi \\ \text{(b) on wall } j \text{ for } x < \xi \end{cases} \\ 1 \quad \text{(a) on wall } j \text{ for } x \geqslant \xi \end{cases}$$

$$\Phi_j^{(3)} = -D_h \frac{\partial\theta_j^{(3)}}{\partial n} = 0,$$

$$\text{(a) on opposite wall for all } x$$

$\theta_j^{(3)}$, *the fundamental solution of the third kind,* corresponds to a solution for a step temperature on one wall, with the other wall remaining insulated.

4. *Fundamental Solutions of the Fourth Kind*

We let $\theta_j^{(4)}(x, r, \Theta; \xi)$ denote the solution of (1) satisfying the boundary conditions

$$\Phi_j^{(4)} = -D_h \frac{\partial\theta_j^{(4)}}{\partial n}$$

$$= \begin{cases} 0 \quad \text{(a) on wall } j \text{ for } x < \xi \\ 1 \quad \text{(a) on wall } j \text{ for } x \geqslant \xi \end{cases}$$

$$\theta_j^{(4)} = 0, \quad \begin{cases} \text{(a) on opposite wall for all } x \\ \text{(b) everywhere upstream of } \xi \end{cases}$$

$\theta_j^{(4)}$, *the fundamental solution of the fourth kind,* corresponds to a solution for a step in heat flux from one wall with the opposite wall maintained at the inlet temperature.

The values of the fundamental solutions and their derivatives on the inner and outer walls are of particular interest. We shall adopt a double subscript notation where the first subscript refers to the wall at which the quantity is evaluated, and the second to the wall at which the non-zero boundary conditions is applied. Thus, for example, $\theta_{io}^{(3)}(x, S_i; \xi)$ represents the value of the fundamental solution of the third kind at the inner wall when the outer wall is wall j. [$\theta_{oo}^{(3)} = 1$ for $x \geqslant \xi$]. Similarly, $\Phi_{oi}^{(1)}(x, S_o; \xi)$ represents the value of $-D_h\ [\partial\theta_i^{(1)}/\partial n]$ evaluated at the outer wall, and is in effect a dimensionless outer wall heat flux corresponding to the fundamental solution of the first kind in which for $x \geqslant \xi$ the inner wall is maintained above and the outer wall is kept at the inlet temperature.

In addition, the mixed-mean values of the fundamental solutions, defined by

$$\theta_{mj}^{(k)}(x; \xi) = \frac{1}{AU_m} \int_A U\theta_j^{(k)}\, dA \qquad (2)$$

are of interest. These represent, in effect, dimensionless mixed-mean temperatures.

* This designation as to "kinds" of solutions was developed by the authors in 1958. Subsequently the same meaning has come into use for the first and second kinds, following a paper by Dzung [6].

USE OF FUNDAMENTAL SOLUTIONS

Examples of simple solutions obtained from the fundamental solutions

Example 1.

Suppose we wish to know the wall heat fluxes for flow in an annulus with inlet temperature t_e, with outer wall maintained at t_e over its entire length, and the inner wall temperature distribution as follows:

$$t_i = t_e, \quad x < \xi,$$
$$t_i = t_I, \quad x > \xi.$$

The solution can be expressed as a constant plus a multiple of a fundamental solution of the first kind, and is, as the reader can easily verify,

$$t(x, r, \Theta) = t_e + (t_I - t_e)\, \theta_i^{(1)}(x, r, \Theta; \xi). \quad (3a)$$

The heat flux from the inner wall would then be

$$q_i''(x, S_i) = -k \left(\frac{\partial t}{\partial n}\right)_i = \frac{k}{D_h}(t_I - t_e)\, \Phi_{ii}^{(1)}(x, S_i; \xi). \quad (3b)$$

The mixed-mean temperature is similarly

$$t_m(x) = t_e + (t_I - t_e)\, \theta_{mi}^{(1)}(x; \xi). \quad (3c)$$

Example 2.

Suppose we wish to know the solution for the inner wall subject to a uniform and constant heat influx q_i'' and the outer wall maintained at temperature t_o from the inlet, where the temperature of the fluid is t_e. The solution can be represented as a linear combination of a constant (t_e), one fundamental solution of the third kind, and one fundamental solution of the fourth kind, as follows:

$$t(x, r, \Theta) = t_e + \frac{q_i'' D_h}{k}\, \theta_i^{(4)}(x, r, \Theta; o) + (t_o - t_e)\, \theta_o^{(3)}(x, r, \Theta; o). \quad (4a)$$

The reader may verify that this indeed satisfies the desired boundary conditions. The temperature distribution on the inner wall is

$$t_i(x, S_i) = t_e + \frac{q_i'' D_h}{k}\, \theta_{ii}^{(4)}(x, S_i; o) + (t_o - t_e)\, \theta_{io}^{(3)}(x, S_i; o) \quad (4b)$$

and the heat influx from the outer wall is†

$$q_o''(x, S_o) = q_i''\, \Phi_{oi}^{(4)}(x, S_o; o) + \frac{k}{D_h}\, \Phi_{oo}^{(3)}(x, S_o; o). \quad (4c)$$

The mixed-mean temperature would then be

$$t_m(x) = t_e + q_i'' \frac{D_h}{k}\, \theta_{mi}^{(4)}(x; o) + (t_o - t_e)\, \theta_{mo}^{(3)}(x; o). \quad (4d)$$

Example 3.

Consider the case where the heat flux is equal to q'' on both walls from the inlet to a point $x = L$, and thereafter the walls are insulated. The inlet temperature is t_e. Of interest are the wall temperatures in both the heated and unheated region. The solution is

$$t(x, r, \Theta) = q'' \frac{D_h}{k} [\theta_i^{(2)}(x, r, \Theta; o) - \theta_i^{(2)}(x, r, \Theta; L) + \theta_o^{(2)}(x, r, \Theta; o) - \theta_o^{(2)}(x, r, \Theta; L)]. \quad (5a)$$

The inner wall temperature distribution would be

$$t_i(x, S_i) = \frac{q'' D_h}{k} [\theta_{ii}^{(2)}(x, S_i; o) - \theta_{ii}^{(2)}(x, S_i; L) + \theta_{io}^{(2)}(x, S_o; o) - \theta_{io}^{(2)}(x, S_i; L)]. \quad (5b)$$

Similar expressions can be developed for t_o and t_m.

In the examples above, the reader will see that the fundamental solutions are, in effect, "influence functions" in the superposition procedure, and there is indeed marked similarity between the present methods and beam deflection analysis. More specific numerical examples are included in subsequent parts.

GENERAL APPROACH FOR ARBITRARILY SPECIFIED WALL TEMPERATURES OR HEAT FLUXES

The superposition technique allows an infinite variety of problems to be handled in a rather

† In verifying this result, recall that n always denotes an inward normal direction.

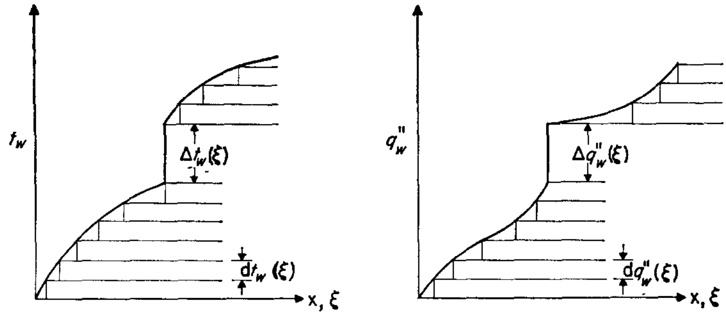

FIG. 2. Superposition of wall temperature and heat flux steps.

general way. Similar methods have been developed for boundary layer flows [7] and for circular tubes [8]. We imagine representing any prescribed wall temperature (or heat flux) variation in terms of the sum of infinitesimal and finite steps, as shown in Fig. 2. Using the four kinds of fundamental solutions, three general solutions can be written, and these are given in Table 1. All integrals appearing in the general solutions are to be evaluated in the Stieltjes sense where finite discontinuities in the prescribed wall temperature or heat flux occur.

Usually one finds that the effects of axial variations in wall temperature or heat flux are only important when the axial gradients are substantial in comparison with the fluid–wall temperature difference [9, 10]. However, asymmetric differences, i.e. different heat fluxes on opposite walls, can produce startlingly important effects. It should be observed that the general solutions are able to handle both axial and asymmetric variations, provided of course that the fundamental solutions and the associated functions are known. At the time of the initiation of this study there was information in the literature pertinent to laminar flow between parallel planes (a limiting annulus) for uses of the first and second kinds. The laminar annulus problem had not been treated at all completely, and existing treatments were not reducible to fundamental solution form. Although a substantial body of turbulent data and some analysis existed, further careful experimental and analytical work was clearly needed, and the present study was directed towards this end.

GENERAL DESCRIPTION OF THE PROGRAM

The central objective of the present study was the evaluation of the four kinds of fundamental solutions. It soon became apparent that this was a goal of such immense scope that it would be impracticable to attain, but it was found that sufficient information could be obtained through combined analytical and experimental studies to allow future prediction in a large variety of important annulus heat-transfer situations.

A complete treatment of the four fundamental solutions for hydrodynamically fully developed laminar flow in concentric annuli was obtained, and will be reported in a subsequent publication [11]. This includes a comprehensive analysis of the eigenvalue problem, as well as asymptotic solutions valid near the boundary condition discontinuities. Experimental data giving excellent verification of the analysis were also obtained.

An analysis of the turbulent heat transfer in concentric annuli for the special case of prescribed heat flux (second kind) with fully-developed velocity profiles was made for a wide range of Prandtl numbers, and confirmed experimentally in tests using air. For $Pr = 0{\cdot}7$ the fundamental solutions of the second kind were determined experimentally and will be reported subsequently [12].

The apparatus employed in the annulus experiments allowed experimental verification of the superposition techniques for asymmetric heating, but did not allow the effects of axial heat flux variation to be studied experimentally. A parallel-planes apparatus was used to study the axial effects for the "unity radius ratio annulus"

Table 1. General solutions

First kind	Second kind	Third kind
$t_j(x)$ Prescribed	$t_j(x) = t_e + \frac{D_h}{k}\int_{\xi=0}^{x}\theta_{jj}^{(2)}(x, S_j; \xi)\,\mathrm{d}q_j''(\xi) + \frac{D_h}{k}\int_{\xi=0}^{x}\theta_{jl}^{(2)}(x, S_j; \xi)\,\mathrm{d}q_l''(\xi)$	$t_j(x)$ Prescribed
$t_l(x)$ Prescribed	$t_l(x) = t_e + \frac{D_h}{k}\int_{\xi=0}^{x}\theta_{lj}^{(2)}(x, S_l; \xi)\,\mathrm{d}q_j''(\xi) + \frac{D_h}{k}\int_{\xi=0}^{x}\theta_{ll}^{(2)}(x, S_l; \xi)\,\mathrm{d}q_l''(\xi)$	$t_l(x) = t_e + \int_{\xi=0}^{x}\theta_{lj}^{(3)}(x, S_l; \xi)\,\mathrm{d}t_j(\xi) + \frac{D_h}{k}\int_{\xi=0}^{x}\theta_{ll}^{(4)}(x, S_l; \xi)\,\mathrm{d}q_l''(\xi)$
$q_j''(x) = \frac{k}{D_h}\int_{\xi=0}^{x}\Phi_{jj}^{(1)}(x, S_j; \xi)\,\mathrm{d}t_j(\xi) + \frac{k}{D_h}\int_{\xi=0}^{x}\Phi_{jl}^{(1)}(x; S_j; \xi)\,\mathrm{d}t_l(\xi)$	$q_j''(x)$ Prescribed	$q_j''(x) = \frac{k}{D_h}\int_{\xi=0}^{x}\Phi_{jj}^{(3)}(x, S_j; \xi)\,\mathrm{d}t_j(\xi) + \int_{\xi=0}^{x}\Phi_{jl}^{(4)}(x, S_j; \xi)\,\mathrm{d}q_l''(\xi)$
$q_l''(x) = \frac{k}{D_h}\int_{\xi=0}^{x}\Phi_{lj}^{(1)}(x, S_l; \xi)\,\mathrm{d}t_j(\xi) + \frac{k}{D_h}\int_{\xi=0}^{x}\Phi_{ll}^{(1)}(x, S_l; \xi)\,\mathrm{d}t_l(\xi)$	$q_l''(x)$ Prescribed	$q_l''(x)$ Prescribed
$t_m(x) = t_e + \int_{\xi=0}^{x}\theta_{mj}^{(1)}(x; \xi)\,\mathrm{d}t_j(\xi) + \int_{\xi=0}^{x}\theta_{ml}^{(1)}(x; \xi)\,\mathrm{d}t_l(\xi)$	$t_m(x) = t_e + \frac{D_h}{k}\int_{\xi=0}^{x}\theta_{mj}^{(2)}(x; \xi)\,\mathrm{d}q_j''(\xi) + \frac{D_h}{k}\int_{\xi=0}^{x}\theta_{ml}^{(2)}(x, \xi)\,\mathrm{d}q_l''(\xi)$	$t_m(x) = t_e + \int_{\xi=0}^{x}\theta_{mj}^{(3)}(x; \xi)\,\mathrm{d}t_j(\xi) + \frac{D_h}{k}\int_{\xi=0}^{x}\theta_{ml}^{(1)}(x; \xi)\,\mathrm{d}q_l''(\xi)$

and the fundamental solutions of all four kinds in turbulent flow were theoretically determined for $Pr = 0{\cdot}7$. A variety of examples were run showing the effects of axial variations and demonstrating the validity of the superposition techniques [13].

The eccentric annulus studies were limited to experimental studies with air, in both laminar and turbulent flow. Some extremely interesting eccentricity effects were observed [14].

The case of simultaneously developing laminar velocity and temperature fields was treated analytically, and experimental verification was obtained for $Pr = 0{\cdot}7$. In addition, some highly significant data regarding the effect of radius ratio on transition were obtained, and compared with an approximate calculation of the critical Reynolds number in developing annular flow [15].

THE ANNULUS APPARATUS

The apparatus employed in the experimental studies of annulus heat transfer is shown in Figs. 3 and 4. The experimental annular tubes were formed from four 6-ft long Inconel tubes. Two outer tubes had nominal inside diameters 1 in and 2 in, and two inner tubes had nominal outside diameters, $\frac{3}{8}$ in and $\frac{1}{2}$ in. Thus, when assembled, four different circular annulus radius ratios could be formed with nominal ratios 0·192, 0·255, 0·375, and 0·500. In addition, some less complete experiments were performed with hypodermic tubing, 0·058 in diameter, mounted inside the 2-in outer tube to give a radius ratio of 0·029.

The tubes were mounted vertically with air flow from the bottom upwards. A nozzle entrance was employed on the outer tube, and then there followed a 2-ft velocity developing section. For turbulent runs, sandpaper boundary layer trips were attached to both inner and outer tubes 4–5 in from the nozzle since it was found that the turbulence level of the entrance system was so low that laminar flow would persist to very high Reynolds numbers, and also transition apparently does not necessarily occur at the same axial position on both surfaces of an annulus.

Heating was accomplished by passing low voltage AC through the lengths of the Inconel tubing. Heating was then at essentially constant heat rate per unit of tube length, and it was not possible to vary heat flux axially, although of course any degree of heating asymmetry could be established (excepting negative heat flux ratios, there being no way to cool the tubes).

The outer tube was supported in three lucite holders, two at the extreme ends of the tube and one at the point of the electrical connection to the heated tube. The core tube was held by two aluminum holders mounted 8 in beyond the ends of the outer tube. To minimize eccentricity the holders were clamped together and bored simultaneously. While clamped, two aligning holes were drilled and reamed through all of the pieces. During the assembly on the supporting channel these aligning holes were brought into line with a transit equipped with a traversing rack. Machined brass plugs with a central hole 0·032 i.d. were pressed into the aligning holes.

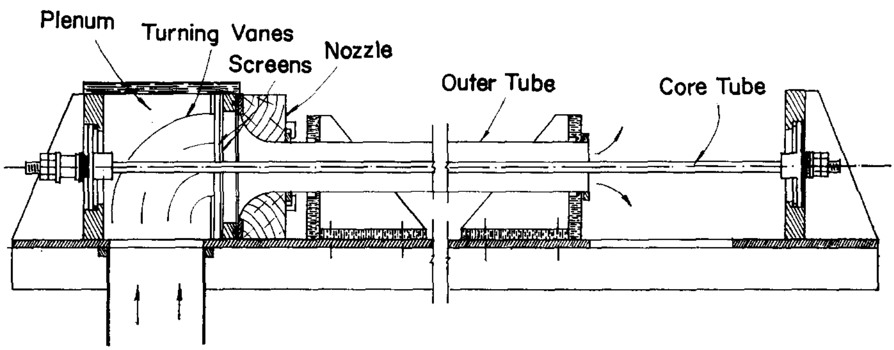

FIG. 3. Experimental annulus assembly.

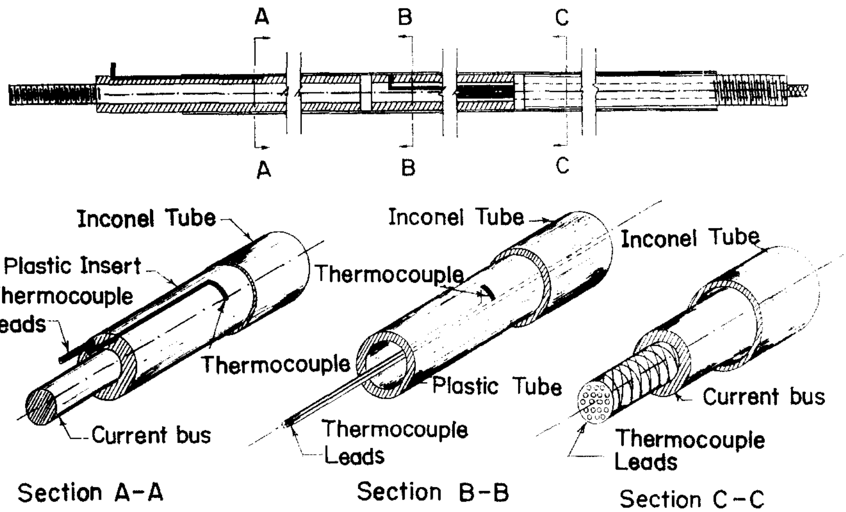

FIG. 4. Core tube thermocouple installation.

These holes were readily distinguished by the transit during the assembly, and it is felt that the axial misalignment of the holders was less than $\pm 0{\cdot}015$ in.

Four brass aligning spheres were made each having a diameter 0·010 in less than the annular gap provided by the four geometries. During the running of the tests the concentricity was periodically checked by lowering the appropriate sphere into the test section from the top. This insured that the axial misalignment was less than 4 per cent of the smallest gap width under all conditions.

In spite of these precautions, some difficulty with eccentricity and poor flow distribution was encountered but the several peripherally located thermocouples provided an effective check on both of these effects. It was ultimately possible to obtain peripheral temperature variations with less than $\frac{1}{2}$ degF maximum differences. (Experiments with an eccentric annulus will be described in a later paper.)

The power expended in the tubes was measured with an ammeter in series with and a high impedence vacuum tube voltmeter across each tube. Local mixed-mean temperatures were deduced from the measured flow rates and power inputs. Power was transmitted to the outer tube through copper bus rings soft-soldered to the outside surface, and to the inner tube through the copper bus rods shown in Fig. 4. In the experiments for hydrodynamically developed flow a 2-ft unheated section was provided in front of the 4-ft heated section. The tubes were reversed for the studies involving simultaneous development of the velocity and temperature fields.

The entire assembly was insulated with several inches of fiberglass. Heat transfer from the inner tube was assumed to be equal to the measured power input to the tube, heat leak to the ends having been determined to be negligible. Heat transfer from the outer tube to the internal air differed from the measured power because of heat leak through the insulation. The magnitude of the heat leak was established by blocking the ends of the tubes and running calibration tests, a procedure that was repeated every time the insulation was re-installed. In turbulent flow, the outer tube heat leak varied from a maximum of 5·8 per cent of the power input at the lowest Reynolds numbers to 0·4 per cent at the highest Reynolds numbers. Somewhat greater heat leaks were encountered in the laminar flow runs.

The temperature measurement system consisted of two thermocouples in the air stream upstream of the nozzle, and then forty-two more on the annulus tube walls. For the $r^* = 0{\cdot}029$ experiments the inner tube was too small to permit use of multiple thermocouples. Instead, a single thermocouple was strung through the tube so that it could be moved up and down the tube to any desired position. All temperatures were made from calibrated 30 gage iron-constant wires. Outer tube thermocouples were spot welded to the tube, wrapped around once to minimize wire temperature gradients, and insulated against heat leak. Core tube thermocouples were fabricated on the outside of a tight fitting plastic insert, as shown in Fig. 4. The copper bus leads were soft soldered into place after insertion of the instrumented insert.

The air entered at the lower end of the supporting channel which was sealed to provide a small chamber 6 in on a side. To facilitate the right angle turn made by the flow a set of sheet metal turning vanes was mounted inside the plenum chamber.* The entrance is a wooden elliptical nozzle. The joint between the nozzle and the tube section was carefully filled, sanded, and varnished to provide a continuous wall surface and to avoid tripping the boundary layer during the laminar flow runs. Screens were placed just upstream of the nozzle to reduce the turbulence level and promote uniform distribution of the flow. The velocity profile leaving the nozzle was measured and found to be slightly dog-eared (7 per cent higher velocity at edge). With a short extension, the velocity profile flattened to within 2 per cent of the centerline velocity. Accordingly it is felt that adequate turning and straightening of the flow was obtained. The turbulence intensity, as measured with a constant temperature linearized hot film anemometer [1] was less than $\frac{1}{2}$ per cent when three screens were used, and of the order of 2 per cent when only two screens were used. The low turbulence level was obtained in all laminar flow experiments. The air, at essentially atmospheric pressure, was provided by a blower and metered with an ASME standard orifice installation. A variety of orifices were used to provide a range for accurate flow measurement (± 1 per cent) of from 1 to 750 cfm.

The 2-ft unheated starting length was deemed sufficient for turbulent flow experiments on the basis of an analysis of Deissler [16]. For a circular tube, the wall shear stress becomes "fully developed" after about five diameters of flow. This analysis assumes a turbulent boundary layer from the inlet, when in actuality a laminar layer will form first. However, it is felt that the addition of heavy tripping established the virtual origin of the turbulent layers within five diameters of the inlet, so that in excess of five diameters were still available for developing the turbulent flow. In the worst case twelve hydraulic diameters were available for entrance length development, with more than this for the radius ratios nearer unity. Although the apparatus was designed primarily for turbulent flow, it was surprisingly possible to get good data in laminar flow. In the Reynolds number ranges of interest the laminar velocity profile is still developing in a tube at twelve diameters. Evidently, however, the small departures form the fully-developed laminar velocity profile did not seriously influence the laminar heat transfer, for good agreement between the experiments and the theory for hydrodynamically developed flow was obtained [11].

Data were reduced via digital computer program. The apparatus and the data reduction techniques are described more fully in references [2], [4], and [5].

* The vanes were removed in later turbulent experiments.

CONCLUDING REMARKS

The general approach to the problem of heat transfer to laminar or turbulent flow in annular passages developed in this paper will be applied and compared with experimental results in the subsequent parts. In general the superposition concept is found to be supported by the data. The effects of asymmetry in the heat flux of wall temperatures is usually significant, while the influence of axial variations on the local convection heat-transfer processes is less pronounced.

REFERENCES

1. W. C. Reynolds, P. A. McCuen, R. E. Lundberg, Y. W. Leung and H. S. Heaton, Heat transfer in annular passages with variable wall temperature and heat flux. Report No. AHT-1, Stanford University, California (1960).
2. P. A. McCuen, W. M. Kays and W. C. Reynolds, Heat transfer with laminar flow in concentric annuli

with constant and variable wall temperature and heat flux. Report No. AHT-2, Stanford University, California (1961).

3. P. A. McCuen, W. M. Kays and W. C. Reynolds, Heat transfer with laminar and turbulent flow between parallel planes with constant and variable wall temperature and heat flux. Report No. AHT-3, Stanford University, California (1962).
4. E. Y. Leung, W. M. Kays and W. C. Reynolds, Heat transfer with turbulent flow in concentric and eccentric annuli with constant and variable heat flux. Report No. AHT-4, Stanford University, California (1962).
5. H. S. Heaton, W. C. Reynolds and W. M. Kays, Heat transfer with laminar flow in concentric annuli with constant heat flux and simultaneously developing velocity and temperature distributions. Stanford University, California (1962).
6. L. S. Dzung, Heat transfer in a flat duct with sinusoidal heat flux distribution. Second U.N. Conference on Atomic Energy, A/Conf. 15/pap 254 (1958).
7. E. R. G. Eckert and R. M. Drake, *Introduction to Heat and Mass Transfer*. McGraw Hill, New York (1957).
8. J. R. Sellars, M. Tribus and J. S. Klein, Heat transfer to laminar flow in a round tube or flat conduit--The Graetz problem extended, *Trans. ASME*, **78**, 441 (1956).
9. W. C. Reynolds, W. M. Kays and S. J. Kline, Heat transfer in the turbulent incompressible boundary layer. III Arbitrary wall temperature and heat flux. *N.A.S.A.* Memo 12-3-58W (1958).
10. W. M. Kays and W. B. Nicoll, The influence of non-uniform heat flux on the convection conductances in a nuclear reactor. Report TR-33, Stanford University, California (1957).
11. R. E. Lundberg, P. A. McCuen and W. C. Reynolds, Heat transfer in annular passages—Hydrodynamically developed laminar flow with arbitrarily prescribed wall temperatures or heat fluxes, *Int. J. Heat Mass Transfer*, to be published.
12. W. M. Kays and E. Y. Leung, Heat transfer in annular passages—Hydrodynamically developed turbulent flow with arbitrarily prescribed heat flux. *Int. J. Heat Mass Transfer* 6, 425 (1963).
13. P. A. McCuen, W. M. Kays and W. C. Reynolds, Heat transfer in annular passages—Hydrodynamically developed turbulent flow between parallel planes with arbitrarily prescribed wall temperature or heat flux. To be published.
14. E. Y. Leung and W. M. Kays, Heat transfer in annular passages—Hydrodynamically developed turbulent flow with eccentricity. To be published.
15. H. S. Heaton, W. C. Reynolds and W. M. Kays, Heat transfer in annular passage—Simultaneous development of velocity in temperature fields in laminar flow. To be published.
16. R. G. Deissler, Analysis of turbulent heat transfer and flow in the entrance region of smooth passages. *N.A.C.A.* TN 3016 (1953).

GENERAL ANNULUS BIBLIOGRAPHY

1. R. V. Bailey, Heat transfer to liquid metals in concentric annuli. Oak Ridge National Laboratory Report ORNL-521 (1950).
2. H. Barrow, Fluid flow and heat transfer in an annulus with a heated core tube. General Discussion on Heat Transfer. *IME* and *ASME*, 1113–1122 (1955).
3. C. Bory and J. Faure, Experimental determination of the influence of eccentricity on the local coefficient of heat transfer in annular passage. *Acad. Sci., Paris*, **249**, No. 17 (1959).
4. F. G. Carpenter, A. P. Colburn, E. M. Schoenborn and A. Wurster, Heat transfer and friction of water in an annular space. *Trans. AIChE*, **41**, 165–187 (1946).
5. R. D. Cess and E. C. Schaffer, Laminar heat transfer between parallel plates with an unsymmetrically prescribed heat flux at the walls. Applied Scientific Research, A **9**, 64 (1960).
6. R. D. Cess and E. C. Schaffer, Summary of laminar heat transfer between parallel plates with unsymmetrical wall temperatures. *J. Aero. Sci.* 538 (1959).
7. E. S. Davis, Heat transfer and pressure drop in annuli. *Trans. AIChE*, **65**, 755–760 (1943).
8. R. G. Deissler and M. F. Taylor, Analysis of fully developed turbulent heat transfer and flow in an annulus with various eccentricities. *N.A.C.A.* TN 3451 (1955).
9. M. Dufinecy and P. Marcus, Heat transfer coefficients in annular spaces. M.S. Thesis, Carnegie Institute of Technology (1938).
10. A. S. Foust and G. A. Christain, Non-boiling heat transfer coefficients in annuli. *Trans. ASME*, **36**, 541–554 (1940).
11. A. S. Foust and T. J. Thompson, Heat transfer coefficients in glass exchangers. *Trans. AIChE*, **36**, 555–575 (1940).
12. W. B. Harrison and J. R. Menke, Heat transfer to liquid metals flowing in asymmetrically heated channels. *Trans. ASME*, **71**, 797–803 (1949).
13. M. Jakob and K. A. Rees, Heat transfer to a fluid in laminar flow through an annular space. *Trans. AIChE*, **37**, 619 (1941).
14. J. G. Knudsden and D. L. Katz, Velocity profiles in annuli. Proceedings of the Midwestern Conference on Fluid Mechanics, First Conference, Illinois (1950).
15. H. Lamb, *Hydrodynamics*. Cambridge University Press, London (1924).
16. A. V. Lisin, W. V. Mackewicz and W. C. Reynolds, Heat transfer in annulus with asymmetric heating. Report No. IDO-28534, Aerojet-General Nucleonics (1959).
17. F. R. Lorenz, On turbulent flow through annular passages, Communications, Institute of Fluid Mechanics, Karlsruhe (1932).
18. W. H. McAdams, *Heat Transmission* (3rd Ed.). McGraw-Hill, New York (1954).
19. E. L. McMillen and R. E. Larson, Annular heat transfer coefficients for turbulent flow. *Trans. AIChE*, **40**, 177–202 (1944).

20. E. L. McMillen and R. E. Larson, Discussion on annular heat transfer coefficients for turbulent flow. *Trans. AIChE*, **41**, 147–152 (1945).
21. R. C. Martinelli, Heat transfer to molten metals. *Trans. ASME*, **69**, 947–959 (1947).
22. T. Migushiva, Analogy between fluid friction and heat transfer in annuli. General Discussion on Heat Transfer. *IME* and *ASME*, 191–192 (1951).
23. P. Miller, J. J. Byrnes and D. M. Benforado, Heat transfer to water in an annulus. *J. AIChE*, **1**, 501 (1955).
24. W. E. Milne, *Numerical Solution of Differential Equations*. Wiley, New York (1953).
25. C. C. Monrad and J. F. Pelton, Heat transfer by convection in annular spaces. *Trans. AIChE*, **38**, 593–611 (1942).
26. A. C. Mueller, Heat transfer from wire to air in parallel flow. *Trans. AIChE*, **38**, 613–629 (1942).
27. K. Murakawa, Analysis of temperature distribution on nonisothermal laminar flow of pipes with annular space. *Trans. Jap. Soc. Mech. Engrs*, **18**, (67), 43 (1952).
28. K. Murakawa, Heat transmission in laminar flow through pipes with annular space. *Trans. Jap. Soc. Mech. Engrs*. **88**, (19), 15 (1953).
29. W. M. Owen, Experimental study of water flow in annular pipes. *Proc. ASCE*, **77**, No. 88 (1951).
30. R. R. Rothfus, C. C. Monrad and V. E. Senecal, Velocity distribution and fluid friction in smooth concentric annuli. *Industr. Engng. Chem.* **42**, 12, 2511–2520 (1950).
31. J. Schenke, A problem of heat transfer in laminar flow between parallel plates. *Appl. Sci. Res.* A **5**, 241 (1955).
32. R. A. Seban and D. F. Casey, Heat transfer to lead-bismuth in turbulent flow in an annulus. *Trans. ASME*, **79**, 1514, 1518 (1957).
33. R. P. Stein, The dependence of the heat transfer coefficient on the ratios of the heat fluxes from the walls of parallel plane flow channel. Reactor Heat Transfer Conference, TID-7529, Part 1 (1956).
34. R. P. Stein and W. Begell, Heat transfer to water in turbulent flow in internally heated annuli. *J. AIChE*, **4**, 127–131 (1958).
35. L. M. Trefethen, Liquid metal heat transfer in circular tubes and annuli. General Discussion on Heat Transfer, *Inst. Mech. Engng* and *ASME*, 436–438 (1951).
36. P. H. Wang, On the convective heat transfer for turbulent flow of single phase fluid through concentric annuli. *Scientia Sinica*, **8**, 98–109 (1959).
37. J. H. Wiegand, Evaluation of some factors affecting material transfer coefficients in annuli. Ph.D. Thesis, University of Michigan (1941).
38. J. H. Wiegand and E. M. Baker, Transfer processes in annuli. *Trans. AIChE*, **38**, 569–592 (1942).
39. A. H. Zerban, Clarification of heat transfer characteristics of fluids in annular passages. Ph.D. Thesis, University of Michigan (1940).

Résumé—Cet article est le premier d'une série couronnant quatre années d'études sur les échanges thermiques dans des conduites annulaires. Il présente une formulation générale du problème, les contributions de plusieurs phases du programme, décrit l'appareillage expérimental utilisé et donne une importante bibliographie.

Zusammenfassung—Die vorliegende Arbeit ist die erste einer Reihe, die über eine vierhährige Forschungstätigkeit an Ringräumen berichten soll. Es wird eine allgemeine Formulierung entwickelt und verschiedene Phasen des Programms werden dargelegt. Die Versuchsapparatur zur Bestimmung des Wärmeüberganges in Ringräumen wird beschrieben und diesbezüglich eine ausführliche Bibliographie angegeben.

Аннотация—Настоящая статья является первой из серии статей, обобщающей результаты четырехлетнего изучения вопроса теплообмена в кольцевых каналах. Дается общая формулировка задачи и вкратце описываются результаты каждого этапа исследования. Дано описание установки, используемой для экспериментального определения параметров теплообмена в кольцевых каналах, а также приведен обширный библиографический материал.